A Comprehensive Design Journey: HyperX Full-Flow Staged Combustion Engine from Concept to 3D Model

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Figure 1: HyperX 3D rendering image. For better visibility, no structural support is added, and components might look bigger in size than the actual geometry or showcase larger space between components

Abstract

This paper presents a comprehensive theoretical framework for designing and developing the HyperX, a full-flow staged combustion (FFSC) rocket engine that utilizes liquid methane (LCH_4) and liquid oxygen (LOX) as propellants. Beginning with fundamental principles of rocket propulsion and thermodynamics, we systematically detail the process of translating theoretical concepts into a complete 3D engine design. The HyperX engine, targeting 1800 kN of thrust at a chamber pressure of 300 bar, represents a methodical approach to modern rocket engine development. We explore the detailed design considerations for each critical component—from the main combustion chamber and nozzle to the turbopump assembly and preburners—providing the mathematical foundations, system design parameters, and geometrical specifications necessary for each element. This paper offers valuable insights into the iterative design workflow that precedes computational fluid dynamics (CFD) analysis and additive manufacturing, serving as both an educational resource and a practical guide for rocket engine development. The HyperX design demonstrates how theoretical principles can be applied to create high-efficiency propulsion systems capable of meeting the demanding requirements of modern space launch vehicles.

1 Introduction

The design and development of rocket engines represents one of the most technically demanding undertakings in aerospace engineering. In recent years, numerous commercial spaceflight organizations have focused on the construction of increasingly efficient propulsion systems. Although the fundamental physical principles governing rocket engine operation remain unchanged, successive design iterations incorporate advanced materials, improved thermodynamic cycles, refined combustion processes, and optimized fluid dynamics to enhance overall performance and efficiency.

Rockets function according to Newton's third law—"For every force, there is an equal and opposite force". The design of the rocket engine involves several physics elements, including thermodynamics, fluid dynamics, classical physics, and related subfields.

Numerous references including *Design Of Liquid Propulsion Engine* by Huang et al., *Rocket Propulsion Elements* by Sutton P et al. talk about the design of rocket engines. Rocket engine designs have a few major steps, and each step has multiple iterations.

1. Theoretical rocket design
2. Computational Fluid Dynamics (CFD)
3. Additive Manufacturing

This paper presents the theoretical design of the rocket, corresponding to the first stage of the development process, namely the transition from conceptual theory to three-dimensional (3D) design. In the following, the rocket engine configuration examined in this work will be referred to as "HyperX."

Terms/Jargon	**Description**
Thrust (T)	The force produced by the engine to propel the rocket, generated by high-velocity exhaust gases.
Specific Impulse (Isp)	A measure of propulsion efficiency, defined as thrust per unit weight flow of propellant (s).
Chamber Pressure (Pc)	The static pressure inside the combustion chamber where fuel, and oxidizer react.
Chamber Temperature (Tc)	The temperature within the combustion chamber after propellant ignition.
Throat Pressure (Pt)	The pressure at the nozzle's narrowest point (throat), influencing exhaust acceleration.
Throat Temperature (Tt)	The temperature of gases at the throat; important for determining sonic conditions.
Exit Pressure (Pe)	The static pressure of the exhaust gases as they exit the nozzle.
Exit Temperature (Te)	The temperature of the exhaust gases at the nozzle exit.
Atmospheric Pressure (Pa)	Ambient pressure outside the rocket, typically sea-level or vacuum reference.
COPV	Composite Overwrapped Pressure Vessel; a lightweight, high-pressure tank for storing pressurants.
Mass flow rate ($\dot{m}$)	The total mass of propellant flowing per second through the engine (kg/s).
Exit Area ($\mathrm{A_e}$)	The cross-sectional area of the nozzle exit, affecting expansion and thrust.
Throat Area ($\mathrm{A_t}$)	The smallest cross-sectional area of the nozzle, where flow reaches sonic velocity.
Characteristic Velocity (C^*)	A performance metric representing combustion efficiency, independent of nozzle geometry.
Thrust Coefficient ($\mathrm{C_f}$)	A factor that relates chamber pressure and nozzle area to actual thrust.
Area Ratio ($\mathrm{A_e/A_t}$)	The ratio of exit area to throat area; a key parameter for nozzle expansion.
Gamma – specific heat ratio ($\mathrm{C_p/C_v}$)	The specific heat ratio $\gamma = \frac{C_p}{C_v}$; affects flow properties and Mach behavior.
Fuel density (ρ_{fuel})	The density of the fuel (e.g., kg/m^3), used in mass and volumetric flow calculations.
Oxidizer density (ρ_{ox})	The density of the oxidizer, similarly used in flow rate and tank sizing.
Spool speed	Rotational speed (RPM) of the turbopump's turbine-pump shaft.

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Terms/Jargon	Description
Pump Power	Mechanical power required to drive fuel or oxidizer through the engine cycle.
Fuel Mass Flow Rate ($\dot{m}_{\text{fuel}}$)	The mass of fuel delivered to the engine per second, typically measured in kilograms per second (kg/s).
Oxidizer Mass Flow ($\dot{m}_{\text{ox}}$)	The mass of oxidizer delivered to the engine per second, also measured in kilograms per second (kg/s).
O/F ratio	Oxidizer-to-fuel mass ratio; critical for combustion performance and thermal balance.
Header tank	A small tank positioned close to the engine, providing stable pressure and propellant flow during startup or transient events.

2 Rocket Engine Cycles

Various rocket engine thermodynamic cycles exist, and these cycles largely determine the corresponding design characteristics of the propulsion system. Some of the most widely employed rocket engine cycles include:

- Full Flow Staged Combustion
- Staged Combustion
- Gas generator
- Expander cycle
- Pressure-fed

The full-flow staged combustion (FFSC) cycle is employed in numerous contemporary liquid-propellant rocket engines, due to its superior thermodynamic efficiency, despite being among the most complex propulsion cycles to design and implement.

HyperX is using the FFSC cycle, and the other engine configurations for HyperX are described below.

Table 2: HyperX Engine Configuration

Parameter	Configuration
Engine Cycle	Full Flow Staged Combustion
Propellant Type	Bi-propellant
Propellant Fuel	Liquid Methane (LCH_4)
Propellant Oxidizer	Liquid Oxygen (LOX)
Engine Type (based on altitude)	Sea level

Rocket design typically begins with the specification of target or nominal values for key performance and structural parameters, followed by the derivation of all secondary design variables from these primary constraints. The principal critical parameters include, but are not limited to, the following:

- Total Thrust (T)
- Chamber pressure (P_c)
- Chamber temperature (P_t)
- Atmospheric Pressure for Engine Type (Sea level or Vacuum) (P_e)

These parameters may require refinement based on computational fluid dynamics (CFD) simulations and additive manufacturing (AM) trials; however, the design process is initiated from theoretically derived baseline values. Rocket engine development is inherently iterative: the theoretical configuration constitutes the initial step, followed by multiple feedback loops in which results from CFD analyses and additive manufacturing experiments inform successive modifications of the theoretical design.

3 Initial requirements

HyperX design begins from a primary sample specification that encapsulates the top level rocket engine configuration. subsequently, this sample specification is propagated throughout all engine subsystems and components, where it serves as the foundational input for deriving, and computing the associated performance parameters and geometric, structural, and operational quantities.

Table 3: Initial Sample Requirement for HyperX

Parameter	Value
Thrust Target	1800 kN or 404 660 lbf
Chamber Pressure	300 bar
Chamber Temperature	3000 K
Atmospheric Pressure (sea level)	1.013 25 bar

Calculations and constants based on initial requirements (with a few assumptions):

Table 4: Constants (based on initial trial values)

Parameter	Value
O/F Mixture ratio	3.0
γ	1.1897
Gas constant R	450
Chamber Efficiency	0.96
Nozzle Efficiency	0.98
g_0 (m/s^2)	9.80665

Table 5: Common Formulas

Parameter	Formula
Exhaust Velocity V_e	$$V_e = \sqrt{\frac{2\gamma}{\gamma-1} RT_c \left[1 - \left(\frac{P_e}{P_c}\right)^{\frac{\gamma-1}{\gamma}}\right]} \quad (3.1)$$ and with molecular mass: $$V_e = \sqrt{\frac{2\gamma}{\gamma-1} \frac{R_u T_c}{\mathfrak{M}} \left[1 - \left(\frac{P_e}{P_c}\right)^{\frac{\gamma-1}{\gamma}}\right]} \quad (3.2)$$
Thrust (F)	$$F = \dot{m} V_e + (P_e - P_a) A_e \quad (3.3)$$
Area Ratio ε	$$\varepsilon = \frac{A_e}{A_t} = \frac{1}{M_e} \left[\frac{2}{\gamma+1} \left(1 + \frac{\gamma-1}{2} M_e^2\right)\right]^{\frac{\gamma+1}{2(\gamma-1)}} \quad (3.4)$$
Characteristic Velocity (c^*)	$$c^* = \sqrt{\frac{RT_c}{\gamma}} \left(\frac{\gamma+1}{2}\right)^{\frac{\gamma+1}{2(\gamma-1)}} \quad (3.5)$$
Thrust Coefficient (C_f)	$$C_f = \sqrt{\frac{2\gamma^2}{\gamma-1} \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}} \left[1 - \left(\frac{P_a}{P_c}\right)^{\frac{\gamma-1}{\gamma}}\right]} \quad (3.6)$$
Specific Impulse (I_{sp})	$$I_{sp} = \frac{C_f \, c^*}{g_0} \quad (3.7)$$
$\dot{m}$ total	$$\dot{m} = \frac{T}{I_{sp} \, g_0} \quad (3.8)$$
$\dot{m}_{\text{ox}}$	$$\dot{m}_{\text{ox}} = \left(\frac{O/F}{1 + O/F}\right) \dot{m}_{\text{total}} \quad (3.9)$$
$\dot{m}_{\text{fuel}}$	$$\dot{m}_{\text{fuel}} = \dot{m}_{\text{total}} - \dot{m}_{\text{ox}} \quad (3.10)$$

4 System Design

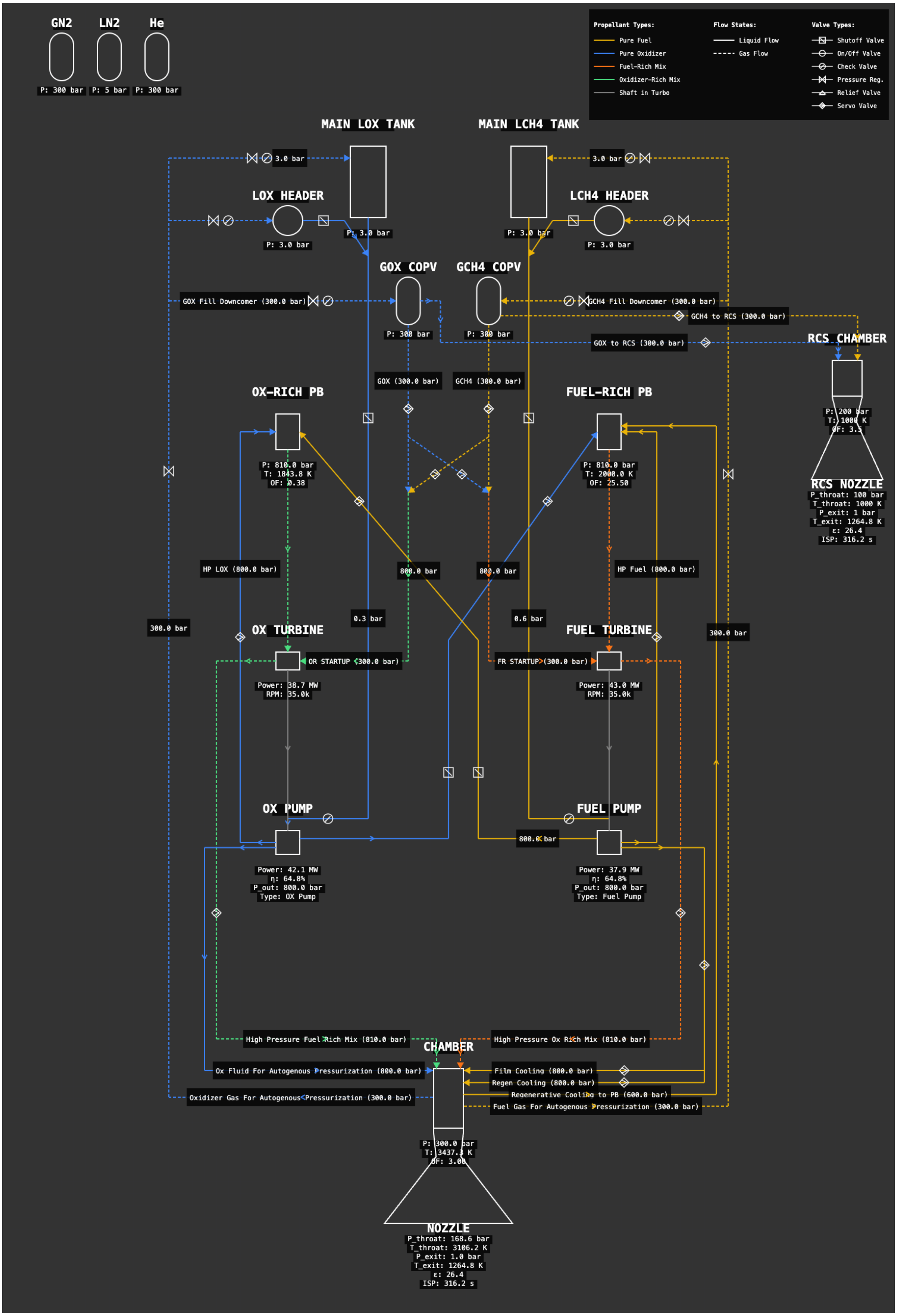

Figure 2: Full system view of HyperX Full Flow Staged Combustion Engine

5 Rocket Engine Elements

A FFSC engine has multiple major components in it. Some of them including,

- Main Combustion Chamber
- Nozzle
- Turbo Pump Assembly (TPA)
- Preburner

In each main component, the system design parameters are calculated theoretically and the geometry is designed based on the system requirements. In each component's system design, the steps to calculate the system design parameters are described and at the end of the system design section, the design values based on HyperX initial requirements are shown in a table.

5.1 Main Combustion Chamber

The main chamber initiates the combustion process by inhibiting the chemical reaction between the fuel and oxidizer in a bi-propellant liquid rocket engine. Its primary functions are **Energy Generation, Pressure Build-up, Uniform Flow Preparation.**

5.1.1 Elements of Main Combustion Chamber

The main combustion chamber has the following sub components,

- Chamber cylinder
- Injector head with multiple coaxial swirl injectors
- Igniter
- Film cooling
- Regenerative cooling

Chamber Cylinder

The chamber cylinder is the primary volume within the combustion chamber where propellants complete mixing and combustion before expansion through the nozzle. It is typically a cylindrical or near-cylindrical section located between the injector face and the nozzle throat.

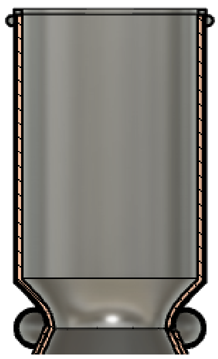

Figure 3: Chamber cylinder cross-section

System Design To estimate thermophysical properties of gas-phase combustion products in a rocket engine, HyperX utilizes NASA's CEA (Chemical Equilibrium with Applications). The following steps outline the process of computing flame temperature, gas constant, specific heat, molecular weight for a given fuel-oxidizer pair, mixture ratio, and chamber pressure.

NASA CEA requires chamber pressure in imperial units (psi). The given chamber pressure in bar is converted accordingly:

$$P_c^{\text{psi}} = P_c^{\text{bar}} \times 14.5037738 \tag{5.1}$$

P_c: is chamber pressure

Effective Flame Temperature:

CEA returns an idealized adiabatic flame temperature. However, in practical applications, this is reduced due to incomplete combustion and the presence of film or regenerative cooling. The effective flame temperature is therefore computed as:

$$T_{\text{flame}} = T_{\text{ideal}} \cdot \eta_{\text{comb}} \cdot \eta_{\text{cooling}} \tag{5.2}$$

$\eta_{comb} \approx 0.96$*: is the combustion efficiency*

$\eta_{cooling} \approx 0.85$*: accounts for thermal losses due to cooling*

T_{flame}*: is the flame temperature calculated from CEA*

T_{ideal}*: is the ideal temperature set in the initial requirement for HyperX*

Molecular Weight:

The molecular weight of the combustion products is obtained from CEA in g/mol and converted to SI units:

$$\text{MW}_{\text{kg/mol}} = \frac{\text{MW}_{\text{g/mol}}}{1000} \tag{5.3}$$

$MW_{g/mol}$*: is the molecular weight acquired from CEA by passing Chamber pressure (psi), mixture ratio, initial expansion ratio of the nozzle. The initial expansion ratio is set as 40 by default if the value is not precalculated*

Mixture Gas Constant:

The specific gas constant of the combustion gas mixture is derived from the universal gas constant and the molecular weight:

$$R_{\text{mix}} = \frac{R_{\text{univ}}}{\text{MW}_{\text{kg/mol}}} \tag{5.4}$$

$R_{univ} = 8.314462618$ ***J/(mol·K)****: is the universal gas constant*

This yields R_{mix} in J/(kg·K), suitable for engineering calculations.

Specific Heat at Constant Pressure (C_p):

The specific heat of the exhaust gases is calculated using the ideal gas relation between γ and R_{mix}:

$$c_p = \frac{\gamma}{\gamma_{\text{exit}} - 1} \cdot R_{\text{mix}} \tag{5.5}$$

γ_{exit}*: is the nozzle exit gamma which is also acquired from CEA along with* $MW_{g/mol}$

This gives the average value of c_p in the nozzle exit region, which is used in turbomachinery and cooling calculations.

Liquid Propellant Density Approximation:

While CEA handles only gas-phase equilibrium, HyperX includes an empirical method to approximate liquid-phase propellant density for pump sizing.

If a reference density ρ_0 at temperature T_0 is known, the liquid density at another temperature T is estimated as:

$$\rho(T) = \rho_0 \cdot [1 - \alpha \cdot (T - T_0)] \tag{5.6}$$

$\alpha \approx 0.0015\, \boldsymbol{K}^{-1}$*: is an approximate thermal expansion coefficient for cryogenic fluids*

If the estimated density becomes unphysically low, it is clamped:

$$\rho(T) = \max\left(\rho(T), 1.0\right) \tag{5.7}$$

Table 6: HyperX Chamber System Values For the Initial Requirement

Parameter	Value
Flame Temperature	3437.25 K
Specific Heat Ratio	1.21
Gas Constant	388.71 J/(kg K)
Molecular Weight	0.02 kg/mol
Specific heat capacity	2212.51 J/(kg K)
Fuel Density	422.63 kg/m^3
Oxidizer Density	1141.00 kg/m^3

Geometry HyperX employs a systematic strategy for computing the main combustion chamber geometry, incorporating both performance-derived parameters (such as characteristic length L^*) and material-stress considerations under high-temperature, high-pressure operation.

Cylindrical Chamber Length from Characteristic Length:

The length of the cylindrical portion of the chamber is derived from the required combustion volume, which in turn is governed by the characteristic length L^*:

$$V_{\text{chamber}} = L^* \cdot A_t \cdot \text{safety\_factor}_{\text{vol}} \tag{5.8}$$

L^*: can be in the range of 0.8..1.2 or up to 1.5. HyperX uses 1.2 as L^* value

Given a desired contraction ratio, the chamber cross-sectional area A_c is:

$$A_c = \text{contraction\_ratio} \cdot A_t \tag{5.9}$$

contraction_ratio: HyperX uses contraction_ratio of 2.5 and the typical values can be in the range of 4..20

The chamber length is then:

$$L_{\text{cyl}} = \frac{V_{\text{chamber}}}{A_c} \tag{5.10}$$

Chamber Radius from Contraction Ratio:

Using the same cross-sectional area, the chamber radius r_c is determined by:

$$A_c = \text{contraction\_ratio} \times \text{throat\_area\_m2} \tag{5.11}$$

$$r_c = \sqrt{\frac{A_c}{\pi}} \tag{5.12}$$

A_c: is the chamber area

Yield Strength Derating Based on Wall Temperature:

Due to material degradation at elevated temperatures, the yield strength is derated above a specified temperature threshold:

$$\sigma_{\text{eff}} = \begin{cases} \sigma_{\text{base}}, & \text{if } T_{\text{wall}} \leq T_{\text{ref}} \\ \sigma_{\text{base}} \cdot f_{\text{derate}}, & \text{if } T_{\text{wall}} > T_{\text{ref}} \end{cases} \tag{5.13}$$

σ_{base}: is the base material yield strength

f_{derate}: is the retained fraction at high temperature

Thermal Wall Thickness from 1D Conduction:

A simplified 1D conduction model can be used to estimate wall thickness required to manage heat flux between hot gas and coolant sides:

$$t_{\text{thermal}} = \frac{k \cdot \Delta T}{q} \tag{5.14}$$

k: is the thermal conductivity of the material

q: is the heat flux [$\mathrm{W/m^2}$]

ΔT: is the allowable temperature gradient across the wall, clamped to a maximum limit

Chamber Wall Thickness (Structural + Thermal):

The wall thickness of the cylindrical section combines structural and thermal requirements. The structural thickness due to internal pressure is:

$$t_{\text{struct}} = \frac{p \cdot r_c}{\sigma_{\text{eff}}/\text{safety\_factor}} \tag{5.15}$$

The total thickness includes the thermal margin and a small manufacturing buffer:

$$t_{\text{total}} = t_{\text{struct}} + t_{\text{thermal}} + 0.001 \tag{5.16}$$

Injector Head

Introduces and atomizes fuel and oxidizer into fine droplets for rapid mixing and combustion. The injector controls combustion stability and performance, using designs such as coaxial, impinging, or swirl injectors.

HyperX uses multiple rings of coaxial swirl injectors with dome manifolds. Each ring has multiple injectors. In each individual injector, oxidizer gas GOX enters from the top and fuel GCH_4 enters the swirl entry on the side.

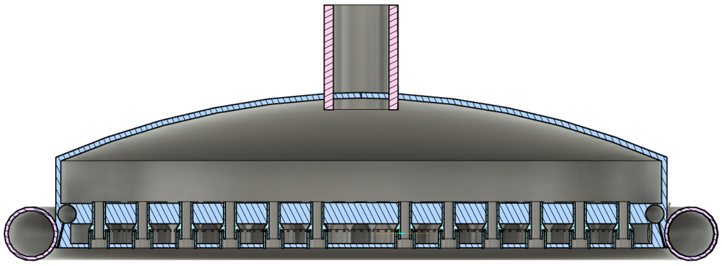

Figure 4: Injector head system design

System Design

Geometry The injector head is designed using a fixed concentric ring layout, with each ring containing a predetermined number of coaxial swirl injectors. The layout and sizing strategy ensure uniform distribution of propellants into the combustion chamber and optimize the performance of the Full-Flow Staged Combustion cycle.

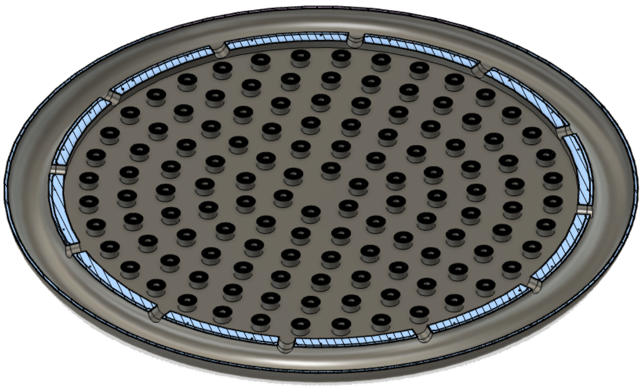

Figure 5: Injector geometry layout

For example, the layout can be [6, 12, 18, 24, 30, 36] injectors → total injectors = sum(list) → 126

Total Injector Count:

The total number of injectors is determined by summing the fixed number of injectors in each concentric circle. This provides a constant and predictable injector arrangement for dome integration.

$$N_{\text{total}} = \sum_{i=0}^{n} N_i \tag{5.17}$$

N_i: represents the number of injectors in the i^{th} ring

Total Propellant Mass Flow Inputs:

The total fuel and oxidizer mass flow rates are provided as part of the system-level engine requirements:

$$\dot{m}_{\text{fuel, total}} = \text{Input parameter} \tag{5.18}$$

$$\dot{m}_{\text{ox, total}} = \text{Input parameter} \tag{5.19}$$

HyperX initial requirement has a fuel flow rate of 145.14 kg/s and oxidizer flow rate of 435.41 kg/s. These are later divided equally among all injectors.

Mass Flow Rate Per Injector:

The fuel and oxidizer mass flow per injector are computed by distributing the total mass flow uniformly across all injectors:

$$\dot{m}_{\text{fuel, per injector}} = \frac{\dot{m}_{\text{fuel, total}}}{N_{\text{total}}} \tag{5.20}$$

$$\dot{m}_{\text{ox, per injector}} = \frac{\dot{m}_{\text{ox, total}}}{N_{\text{total}}} \tag{5.21}$$

This ensures each coaxial swirl injector receives identical flow conditions for combustion stability and mixing efficiency.

Ring Radius Calculation:

The injectors are placed in concentric rings around the central axis of the dome. The radius of each ring is calculated based on the injector outer diameter D_o and a fixed inter-ring spacing value. The radius for the i^{th} ring is defined as:

$$r_i = i \cdot (D_o + s) \tag{5.22}$$

r_i: is the radius of the i^{th} ring

D_o: is the outer diameter of the injector

s: is the fixed spacing between rings

$i = 1, 2, 3, \ldots$: ring index

The first ring ($i = 0$) represents the single central injector located at the origin.

Angular Positioning of Injectors:

Each ring i contains N_i injectors placed symmetrically. The angular position θ_j of each injector j in ring i is given by:

$$\theta_j = \frac{2\pi j}{N_i} \quad \text{for} \quad j = 0, 1, 2, \ldots, N_i - 1 \tag{5.23}$$

This ensures uniform azimuthal spacing around each ring.

This design can be iterated in CFD or additive manufacturing tests by rearranging the injectors in each ring or changing the mass flow of individual injectors.

5.1.2 Coaxial Swirl Injector Single Element Geometry

Each injector element in the engine is modeled as a coaxial swirl injector, designed to ensure efficient atomization and mixing of propellants at the entry to the combustion chamber. The injector geometry is defined using a fixed set of parameters, as outlined below. These parameters are applied identically to all injectors in the dome array.

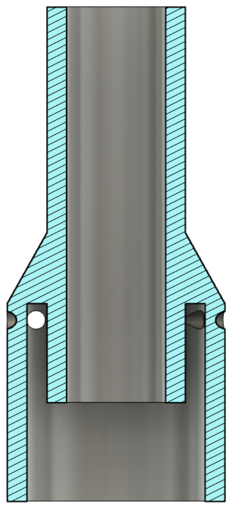

Figure 6: Coaxial swirl injector single element geometry

Inner Diameter (D_i): The diameter of the central injector channel, governing the core flow area and influencing exit velocity and pressure drop.

Outer Diameter (D_o): The total external diameter of the injector element, used for determining spatial layout and packing density on the chamber dome.

Gap Thickness ($t_{\mathbf{gap}}$): The radial gap between coaxial flow channels, often related to wall thickness or the insulation requirements between fuel and oxidizer flows.

Swirl Port Count: The number of tangential ports used to induce swirl in the entering propellant. Higher counts increase swirl intensity and improve atomization.

Swirl Port Diameter: The diameter of each individual tangential port, affecting the total flow area and swirl strength.

Swirl Port Angle: The inclination angle of the swirl ports relative to the injector axis, balancing swirl strength with axial momentum.

Swirl Chamber Diameter: The diameter of the chamber where swirling flow develops before exiting, typically sized as a multiple of the injector's inner diameter.

Exit Cone Angle: The angle of the conical exit section that shapes the spray pattern into the combustion chamber, affecting penetration and mixing.

Igniter

Provides the initial thermal energy to initiate combustion, typically using pyrotechnic devices or augmented torch igniters. Reliable ignition is critical to prevent hard starts or combustion instabilities.

Cooling

Cooling is necessary to manage extreme thermal loads exceeding material limits, ensuring chamber longevity and structural integrity. There are two types of cooling used in HyperX,

- Film Cooling
- Regenerative Cooling

Film Cooling In the HyperX engine architecture, film cooling is employed as a critical supplementary technique to protect the inner chamber walls and nozzle throat from the extreme thermal loads generated during combustion. Approximately **1.2% of the total fuel flow** is diverted after the main fuel pump, maintained at **800 bar**, and routed through a dedicated manifold to deliver high-pressure liquid methane directly into the combustion chamber at strategic locations. These injection points are positioned near the **upper chamber sidewall** and immediately **upstream of the nozzle throat**, where convective heat flux is most intense and regenerative cooling alone may be insufficient.

The injected methane forms a thin, cold boundary layer along the inner wall, effectively shielding the metallic structure from the 3400+ K combustion gases. This protective film layer reduces the net heat transfer into the wall, delays the local thermal degradation of chamber materials, and enables operation at higher chamber pressures without compromising structural integrity. The film cooling flow is not recovered; it becomes part of the combustion process after fulfilling its thermal shielding function. The system is pressure-balanced to ensure consistent film adherence and minimize flow separation or penetration errors. This design offers a high-efficiency, low-mass method of thermal protection, particularly in the throat region where engine performance is highly sensitive to wall temperature gradients.

Regenerative Cooling Circulates propellant through embedded channels to absorb heat. This method cools the walls while preheating the fuel, enhancing overall engine performance and supporting reusability. In the HyperX FFSC engine, high pressure (around 800 bar) fuel from the turbopump outlet is passed through the vertically designed channels on the chamber/nozzle walls to absorb heat via conduction. The fuel is then redirected to the fuel rich preburner to follow the usual flow path.

Figure 7: Regenerative cooling system schematic

System Design <Formulas>
<Calculations>

Figure 8: Regenerative cooling channel design

Geometry HyperX uses the following theoretical procedure for the cooling channels that are passing through the main chamber and the nozzle,

- **Surface Area Calculation:** The total internal hot surface area is computed by summing the areas of:
  - The cylindrical chamber section,
  - The convergent frustum (from chamber radius to throat radius),
  - The throat band (a short cylindrical region around the throat),
  - The divergent frustum (from throat radius to nozzle exit radius).
- **Heat Flux and Total Heat Load:** An average heat flux is estimated for each region, and the corresponding heat load is calculated. The total heat load, Q, is the sum of all regional heat loads.
- **Theoretical Coolant Mass Flow:** The required coolant mass flow rate, $\dot{m}$, is computed to ensure that the coolant outlet temperature T_{out} does not exceed a specified maximum limit:

$$\dot{m} = \frac{Q}{c_p(T_{\text{out}} - T_{\text{in}})} \tag{5.24}$$

c_p: is the coolant specific heat capacity

T_{in}: is the coolant inlet temperature

T_{out}: is set to the allowable maximum

- **Cooling Channel Design Iteration:** Possible channel configurations (number of channels and channel widths) are iterated to identify a feasible design where:
  - Pressure drop across the cooling passage is less than Δp_{max},
  - Coolant velocity is less than v_{max}.
- **Final Solution Selection:** The final cooling solution is selected based on satisfying thermal and hydraulic constraints, returning the optimized channel geometry and required coolant flow rate.

5.2 Nozzle

The rocket nozzle expands and accelerates the hot gases from the combustion chamber to generate thrust. Its primary functions are **Energy Conversion, Flow Acceleration, Pressure Matching.**

5.2.1 Elements of Nozzle

The nozzle has the following key components:

- Convergent section (transition from chamber to throat)
- Throat (where flow becomes sonic)
- Divergent section (bell-shaped or conical expansion)

System Design

The following section outlines the core equations used to estimate the performance characteristics of an ideal gas rocket nozzle under isentropic assumptions. These calculations assume steady, one-dimensional flow with negligible viscous losses and full thermodynamic equilibrium.

Characteristic Velocity:

The characteristic velocity c^* quantifies the combustion performance independently of nozzle expansion. It is calculated using the following expression, assuming ideal gas behavior and isentropic conditions:

$$c^* = \sqrt{\frac{R \cdot T_c}{\gamma}} \cdot \left(\frac{\gamma + 1}{2}\right)^{\frac{\gamma+1}{2(\gamma-1)}} \tag{5.25}$$

R: is the specific gas constant [J/(kgK)]

T_c: is the chamber (stagnation) temperature [K]

γ: is the ratio of specific heats

Thrust Coefficient:

The thrust coefficient relates the actual thrust force to the chamber pressure and throat area. For an ideally expanded nozzle (i.e., $P_e = P_a$), it is given by:

$$C_f = \sqrt{\frac{2 \cdot \gamma^2}{\gamma - 1} \cdot \left(\frac{2}{\gamma + 1}\right)^{\frac{\gamma+1}{\gamma-1}} \cdot \left[1 - \left(\frac{P_a}{P_c}\right)^{\frac{\gamma-1}{\gamma}}\right]} \tag{5.26}$$

P_c: is the chamber pressure [Pa]

P_a: is the ambient pressure [Pa]

Specific Impulse:

Specific impulse is a measure of engine efficiency, defined as thrust per unit propellant flow rate. It is directly related to the product of the thrust coefficient and characteristic velocity:

$$I_{sp} = \frac{C_f \cdot c^*}{g_0} \tag{5.27}$$

g_0: is standard gravity (9.80665 m/s^2)

Total Mass Flow Rate:

The required total mass flow rate to produce the target thrust is derived from the specific impulse:

$$\dot{m} = \frac{T}{I_{sp} \cdot g_0} \tag{5.28}$$

T: is the target thrust [N]

Throat Area:

The throat area is calculated based on the choked flow condition at Mach 1 and is derived by rearranging the isentropic mass flow rate equation:

$$A_t = \frac{\dot{m}}{P_c \cdot \sqrt{\frac{\gamma}{R \cdot T_c}} \cdot \left(\frac{\gamma+1}{2}\right)^{-\frac{\gamma+1}{2(\gamma-1)}}} \tag{5.29}$$

This expression ensures sonic conditions at the throat.

Exit Mach Number:

The exit Mach number is determined numerically by solving the isentropic pressure ratio equation:

$$\frac{P_e}{P_c} = \left[1 + \frac{\gamma - 1}{2} \cdot M_e^2\right]^{-\frac{\gamma}{\gamma-1}} \tag{5.30}$$

M_e: is the exit Mach number

P_e: is the exit pressure [Pa]

This equation is solved using a bisection method to find M_e corresponding to a given exit pressure P_e.

Nozzle Area Ratio:

The area expansion ratio is computed from the exit Mach number using the isentropic area-Mach relation:

$$\frac{A_e}{A_t} = \frac{1}{M_e} \cdot \left[\left(\frac{2}{\gamma+1}\right) \cdot \left(1 + \frac{\gamma - 1}{2} \cdot M_e^2\right)\right]^{\frac{\gamma+1}{2(\gamma-1)}} \tag{5.31}$$

Exit Area:

Once the expansion ratio is known, the exit area is simply:

$$A_e = \left(\frac{A_e}{A_t}\right) \cdot A_t \tag{5.32}$$

Throat and Exit Diameters:

The diameters of the throat and exit sections are obtained from their respective areas:

$$D_t = 2 \cdot \sqrt{\frac{A_t}{\pi}}, \quad D_e = 2 \cdot \sqrt{\frac{A_e}{\pi}} \tag{5.33}$$

Exit Pressure and Temperatures:

The exit static pressure is derived using the isentropic relation at Mach M_e:

$$P_e = P_c \cdot \left[1 + \frac{\gamma - 1}{2} \cdot M_e^2\right]^{-\frac{\gamma}{\gamma-1}} \tag{5.34}$$

Similarly, the temperatures at the throat and exit are:

$$T_t = T_c \cdot \frac{2}{\gamma + 1}, \quad T_e = \frac{T_c}{1 + \frac{\gamma-1}{2} \cdot M_e^2} \tag{5.35}$$

Throat Pressure:

The static pressure at the throat is given by the isentropic Mach 1 condition:

$$P_t = P_c \cdot \left(\frac{2}{\gamma + 1}\right)^{\frac{\gamma}{\gamma-1}} \tag{5.36}$$

Propellant Mass Flow Split:

Given an overall mixture ratio O/F, the fuel and oxidizer mass flow rates are computed as:

$$\dot{m}_{ox} = \frac{O/F}{1 + O/F} \cdot \dot{m}_{total}, \quad \dot{m}_{fuel} = \dot{m}_{total} - \dot{m}_{ox} \tag{5.37}$$

This allows performance mapping onto the hardware-level requirements of the engine injector and turbopump systems.

Table 7: HyperX Nozzle System Values For the Initial Requirement

Parameter	Value
Specific Impulse	316.17 s
Total Mass Flow Rate	580.55 kg/s
Fuel Flow Rate	145.14 kg/s
Oxidizer Flow Rate	435.41 kg/s
Throat Area	0.03 m^2
Throat Diameter	0.21 m
Exit Area	0.91 m^2
Exit Diameter	1.08 m
Exit Mach	4.01
Expansion Ratio (A_e/A_t)	26.44
Thrust Coefficient	1.75
Characteristic Velocity	1775.38 m/s
Throat Pressure	168.58 bar
Exit Pressure	1.01 bar
Throat Temperature	3106.23 K
Exit Temperature	1264.78 K
Flow Expansion	Ideal ($P_e = P_a$)

Shape of the Nozzle

HyperX uses an ideal bell shaped nozzle implemented using the Prandtl-Meyer Method. The nozzle geometry can be divided into convergent and divergent sections.

Figure 9: Nozzle cross-section showing convergent and divergent sections

Convergent Section HyperX uses a two-segment approach for optimal smoothness and manufacturability of the convergent section. The first segment is a cubic polynomial curve that smoothly decreases the chamber radius with a changing slope from the chamber wall toward an intermediate radius. The second segment is a circular arc, carefully blended to control curvature near the throat, ensuring a smooth transition into the nozzle without sharp corners.

Geometry To construct a physically realistic and manufacturable convergent section for the combustion chamber/nozzle, a two-segment approach is used. This avoids abrupt slope transitions and allows control over the curvature, which is critical for thermal and structural optimization near the throat.

Segment 1: Cubic Polynomial Transition

The first segment spans from the chamber radius to an intermediate radius before the throat. It is defined using a cubic polynomial:

$$y(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3 \tag{5.38}$$

This polynomial is constrained using four boundary conditions:
1. The radius at the start of the chamber (at $x = 0$):

$$y(0) = a_0 = r_{\text{chamber}} \tag{5.39}$$

2. The slope at the start of the chamber:

$$y'(0) = a_1 = \text{slope}_{\text{chamber}} \tag{5.40}$$

3. The radius at the intermediate point $x = L_1$, set as the average of the chamber and throat radii:

$$y(L_1) = r_{\text{mid}} = \frac{r_{\text{chamber}} + r_{\text{throat}}}{2} \tag{5.41}$$

4. The slope at the intermediate point is assumed flat:

$$y'(L_1) = 0 \tag{5.42}$$

Substituting into the cubic polynomial and its derivative gives two equations for the remaining coefficients a_2 and a_3:

$$a_0 + a_1 L_1 + a_2 L_1^2 + a_3 L_1^3 = r_{\text{mid}} \tag{5.43}$$

$$a_1 + 2a_2 L_1 + 3a_3 L_1^2 = 0 \tag{5.44}$$

Solving this system provides a smooth transition from the chamber to the intermediate radius.

Segment 2: Circular Arc Toward the Throat

The second segment is a circular arc connecting the mid-radius point (L_1, r_{mid}) to the throat point (L_2, r_{throat}). If the arc radius R is not provided, it is estimated using the chord length between the two endpoints:

$$\text{chord}_{\text{len}} = \sqrt{(L_2 - L_1)^2 + (r_{\text{throat}} - r_{\text{mid}})^2} \tag{5.45}$$

$$R = 0.75 \cdot \text{chord}_{\text{len}} \tag{5.46}$$

The center of the circle (x_c, y_c) is computed geometrically. Let the chord midpoint be:

$$x_m = \frac{L_1 + L_2}{2}, \quad y_m = \frac{r_{\text{mid}} + r_{\text{throat}}}{2} \tag{5.47}$$

Then, the offset h from the midpoint to the arc center is:

$$h = \sqrt{R^2 - \left(\frac{\text{chord}_{\text{len}}}{2}\right)^2} \tag{5.48}$$

A unit normal vector to the chord is:

$$\hat{n} = \left(\frac{-(r_{\text{throat}} - r_{\text{mid}})}{\text{chord}_{\text{len}}}, \frac{L_2 - L_1}{\text{chord}_{\text{len}}}\right) \tag{5.49}$$

And the center coordinates become:

$$x_c = x_m + h \cdot \hat{n}_x, \quad y_c = y_m + h \cdot \hat{n}_y \tag{5.50}$$

Parametric Definition of Arc Profile

Once the arc center is known, the start and end angles are defined as:

$$\theta_1 = \tan^{-1}\left(\frac{r_{\text{mid}} - y_c}{L_1 - x_c}\right), \quad \theta_2 = \tan^{-1}\left(\frac{r_{\text{throat}} - y_c}{L_2 - x_c}\right) \tag{5.51}$$

Then, the arc is sampled uniformly in angle:

$$x_i = x_c + R \cdot \cos(\theta_i), \quad y_i = y_c + R \cdot \sin(\theta_i) \tag{5.52}$$

where $\theta_i \in [\theta_1, \theta_2]$ is linearly spaced over the number of points.

Divergent Section HyperX uses an ideal bell shaped nozzle for the divergent section. In case of ideal bell nozzles, the shape of the bell can be calculated using MoC (Method of Characteristics). MoC is a mathematical technique used to design the contour of a rocket nozzle's divergent section for supersonic, shock-free expansion.

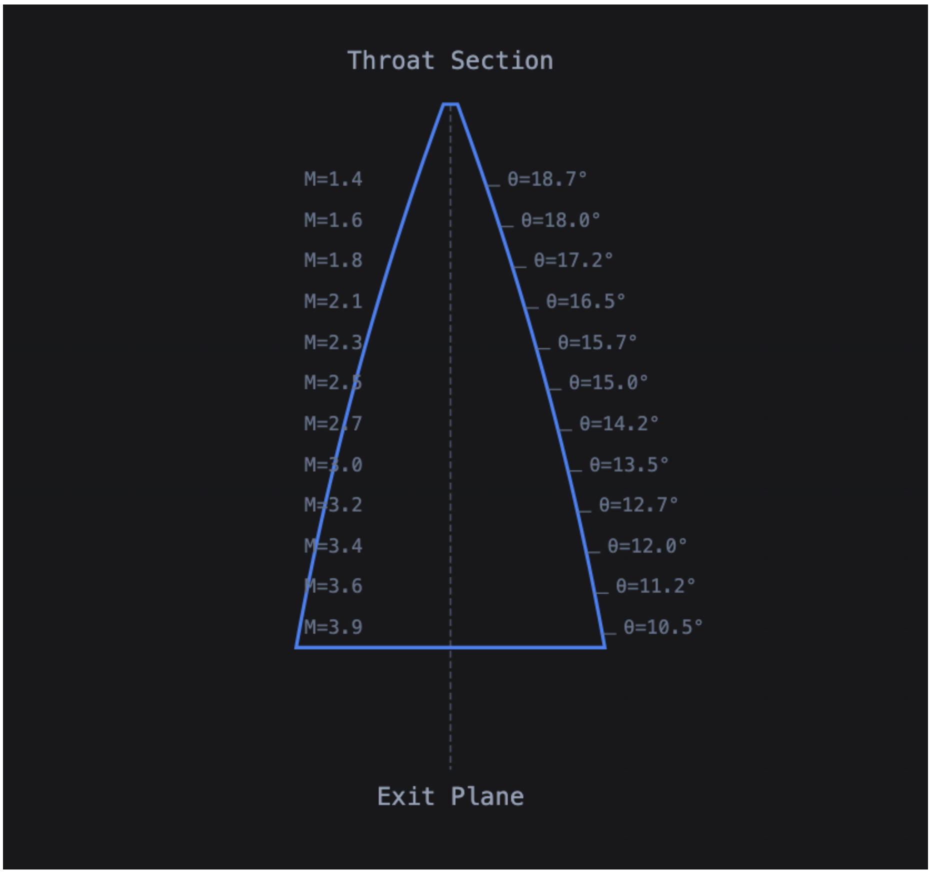

Figure 10: Sample MoC visual with Mach and angle

Geometry To generate an ideal two-dimensional axisymmetric supersonic nozzle profile, a tailored Method of Characteristics (MoC) algorithm is implemented. This method iteratively propagates flow properties from the throat to the exit while satisfying three essential boundary conditions:

- The **exit Mach number** M_{exit}
- The **exit flow angle** θ_{exit}
- The **exit area** A_{exit}, converted into an equivalent exit radius

The solver incrementally ramps both the boundary Mach number and turning angle from throat values to their specified targets. The solution process stops when the nozzle boundary reaches the target radius corresponding to the exit area.

Exit Radius from Area:

To convert the specified exit area into a geometric radius, the following relation is used:

$$r_{\text{exit}} = \sqrt{\frac{A_{\text{exit}}}{\pi}} \tag{5.53}$$

This target radius determines the stopping condition for the characteristic expansion.

Incremental Boundary Updates:

The boundary Mach number is linearly increased from its throat value $M = 1$ to the desired exit Mach number M_{exit} over a fixed number of rows N_{rows}. The incremental step in Mach number is computed as:

$$\Delta M = \frac{M_{\text{exit}} - 1.0}{N_{\text{rows}}} \tag{5.54}$$

Simultaneously, the boundary turning angle is ramped from the initial chamber wall corner angle θ_{corner} to the exit flow angle θ_{exit}. The incremental step in radians is:

$$\Delta\theta = \frac{\theta_{\text{exit}} - \theta_{\text{corner}}}{N_{\text{rows}}} \tag{5.55}$$

At each iteration i, the boundary values are updated as:

$$M_i = M_{i-1} + \Delta M \tag{5.56}$$

$$\theta_i = \theta_{i-1} + \Delta\theta \tag{5.57}$$

Prandtl-Meyer Expansion Function:

The turning angle ν associated with a given Mach number is calculated using the Prandtl-Meyer function:

$$\nu(M) = \sqrt{\frac{\gamma+1}{\gamma-1}} \cdot \tan^{-1}\left(\sqrt{\frac{\gamma-1}{\gamma+1}(M^2-1)}\right) - \tan^{-1}\left(\sqrt{M^2-1}\right) \tag{5.58}$$

This function is evaluated at each boundary point to track the flow expansion state.

Boundary Node Coordinates:

Each new boundary node is placed in the axial direction using a fixed grid increment dx. The axial and radial coordinates are computed as:

$$x_i = i \cdot dx \tag{5.59}$$

$$y_i = y_{i-1} + dx \cdot \tan(\theta_i) \tag{5.60}$$

Interpolation to Match Exact Exit Radius:

If the current boundary node radius y_i exceeds the target radius r_{exit}, linear interpolation is performed between the previous boundary node A and the current node B to precisely meet the exit constraint.

Let: $(x_A, y_A), M_A, \theta_A$ and $(x_B, y_B), M_B, \theta_B$ be the previous and current node properties. The interpolation fraction is:

$$f = \frac{r_{\text{exit}} - y_A}{y_B - y_A} \tag{5.61}$$

Interpolated values for the final boundary node are computed as:

$$x_{\text{final}} = x_A + f \cdot (x_B - x_A) \tag{5.62}$$

$$M_{\text{final}} = M_A + f \cdot (M_B - M_A) \tag{5.63}$$
$$\theta_{\text{final}} = \theta_A + f \cdot (\theta_B - \theta_A) \tag{5.64}$$
$$\nu_{\text{final}} = \nu(M_{\text{final}}) \tag{5.65}$$

This interpolation ensures that both the exit flow angle and the exit radius (hence area) are satisfied to within floating-point accuracy.

Interior Node Computation via Characteristic Intersection:

Interior MoC nodes are computed by intersecting C+ and C characteristic lines originating from adjacent upstream points.

Let the flow angles and Prandtl-Meyer values from the C+ and C points be: θ^+, ν^+ and θ^-, ν^-

We define:

$$K^+ = \theta^+ + \nu^+ \tag{5.66}$$
$$K^- = \theta^- - \nu^- \tag{5.67}$$

The average turning angle and Prandtl-Meyer angle at the intersection are:

$$\theta_* = \frac{K^+ + K^-}{2} \tag{5.68}$$
$$\nu_* = \frac{K^+ - K^-}{2} \tag{5.69}$$

The corresponding Mach number is computed by inverting the Prandtl-Meyer function:

$$M_* = \nu^{-1}(\nu_*) \tag{5.70}$$

The coordinates of the new intersection node are estimated using the midpoint of the source points and a directional step of size dx:

$$x_* = \frac{x^+ + x^-}{2} + dx \cdot \cos(\theta_*) \tag{5.71}$$
$$y_* = \frac{y^+ + y^-}{2} + dx \cdot \sin(\theta_*) \tag{5.72}$$

This approach enables high-fidelity propagation of characteristics through the nozzle domain.

5.3 Turbo Pump Assembly

HyperX has two turbo pump units, one for fuel and another one for oxidizer. Fuel turbo pump is driven by fuel-rich preburner and ox pump is driven by ox-rich preburner. Its primary functions are to pressurize and deliver the propellants to the combustion chamber.

5.3.1 Elements of Turbo Pump Assembly

Each turbo pump unit has the following sub components:

- Turbine: Converts hot gas energy to mechanical shaft work

- Impeller: Pressurizes propellant via centrifugal force
- Inducer: Pre-pressurizes fluid to prevent cavitation before impeller
- Shaft: Transfers torque between turbine and pumps
- Labyrinth seal with swirl breaks: Reduces leakage and swirl along the shaft
- Roller element bearing: Supports radial loads on the shaft
- Ball bearing: Supports axial or combined loads on the shaft
- Secondary flow paths: Channels for balancing, sealing, or cooling flows

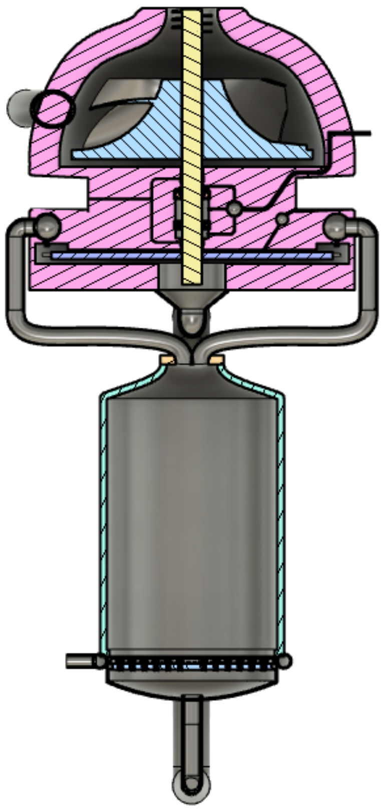

Figure 11: Turbo Pump Assembly schematic

System Design

In a Full Flow Staged Combustion (FFSC) engine like HyperX, the turbopump assembly is critical for delivering propellants at the required pressure and flow rate to the combustion chamber. The fuel turbopump is driven by a fuel-rich preburner, while the oxidizer turbopump is driven by an oxidizer-rich preburner. This design maximizes efficiency by routing all propellants through the turbines before they reach the main combustion chamber.

Hot Gas Properties Calculation Before designing the turbopump assembly, we need to calculate the hot gas properties from the preburner that will drive the turbine. These properties are essential for determining turbine performance and designing the labyrinth seal.

Gas Density Calculation:

The gas density is calculated using the ideal gas law:

$$\rho_{gas} = \frac{P}{R \times T} \tag{5.73}$$

ρ_{gas}: is the gas density [kg/m³]
P: is the gas pressure [Pa], converted from bar using a factor of 10^5
R: is the gas constant [J/(kg·K)] specific to the preburner gas composition
T: is the gas temperature [K] from the preburner

Gas Viscosity Calculation:

The gas viscosity, which affects seal performance and turbine efficiency, is estimated using an empirical relation:

$$\mu = A \times T^{0.7} \tag{5.74}$$

μ: is the gas viscosity [Pa·s]
A: is a reference scale constant determined from preburner gas composition
T: is the gas temperature [K]

This gas viscosity can also be retrieved from NASA CEA.

Pump Power Calculation The pump head is derived from the pressure difference between outlet and inlet:

Pump Head Calculation:

$$H = \frac{(P_{outlet} - P_{inlet}) \times 10^5}{\rho \times g} \tag{5.75}$$

P_{outlet}: is the pump outlet pressure [bar]
P_{inlet}: is the pump inlet pressure [bar]
ρ: is the fluid density [kg/m³]
g: is the gravitational acceleration [9.81 m/s²]
10^5: is the factor to convert bar to Pascal

Total Pump Power Calculation:

The total pump power required is calculated based on the mass flow rate, head rise, and pump efficiency:

$$total\_pump\_power = \frac{\dot{m} \times g \times H}{\eta_{pump}} \tag{5.76}$$

$\dot{m}$: is the mass flow rate of propellant [kg/s]
H: is the pump head [m]
η_{pump}: is the pump efficiency [dimensionless]

Spool Speed Optimization The spool speed significantly impacts the turbopump's performance. HyperX determines the optimal spool speed through iteration, evaluating different speeds to maximize efficiency:

$$\omega = \frac{2\pi \times spool\_speed\_rpm}{60} \tag{5.77}$$

ω: is the angular velocity [rad/s]
spool_speed_rpm: is the rotational speed in revolutions per minute

Pump Efficiency Calculation To model realistic performance, the pump efficiency varies with spool speed and the number of stages:

$$\eta_{pump} = \eta_{base} - penalty + stage\_gain \tag{5.78}$$

η_{base}: is the base pump efficiency determined from fluid properties and pump type
penalty: is the efficiency reduction due to deviation from optimal speed
stage_gain: is the efficiency improvement from multiple stages

Efficiency Penalty Calculation:

The penalty is calculated as:

$$penalty = K_p \times (spool\_speed\_rpm - optimal\_speed)^2 \tag{5.79}$$

K_p: is a penalty coefficient derived from empirical data
optimal_speed: is the speed at which maximum efficiency is achieved

Stage Gain Calculation:

The stage gain is calculated as:

$$stage\_gain = K_s \times (stage\_count - 1) \tag{5.80}$$

K_s: is a stage gain coefficient derived from empirical data
stage_count: is the number of pump stages

This efficiency is constrained within realistic bounds:

$$\eta_{min} \leq \eta_{pump} \leq \eta_{max} \tag{5.81}$$

Pump Stage Design For multi-stage pump configurations, the total head is distributed equally across all stages:

$$stage\_head = \frac{total\_head}{stage\_count} \tag{5.82}$$

total_head: is the total pump head required [J/kg]
stage_count: is the number of pump stages

Ideal Stage Power Calculation:

Using the stage head, the ideal power required for each stage is:

$$ideal\_stage\_power = \dot{m} \times stage\_head \tag{5.83}$$

$\dot{m}$: is the mass flow rate of propellant [kg/s]
stage_head: is the head for each stage [J/kg]

Actual Stage Power Calculation:

The actual power required by each stage, accounting for efficiency losses:

$$actual\_stage\_power = \frac{ideal\_stage\_power}{\eta_{pump}} \tag{5.84}$$

η_{pump}: *is the pump efficiency [dimensionless]*
ideal_stage_power: is the ideal power required [W]

Turbine Efficiency Calculation Similar to the pump, the turbine efficiency varies with operating conditions:

$$\eta_{turbine} = \eta_{base} + speed\_term + stage\_term \tag{5.85}$$

η_{base}: *is the base turbine efficiency for the specific design*
speed_term: is the efficiency adjustment based on spool speed
stage_term: is the efficiency adjustment based on stage count

Speed Term Calculation:

The speed term is calculated as:

$$speed\_term = K_t \times (spool\_speed\_rpm - reference\_speed) \tag{5.86}$$

K_t: *is a turbine speed coefficient derived from empirical data*
reference_speed: is a baseline speed for efficiency calculations

Stage Term Calculation:

The stage term is calculated as:

$$stage\_term = K_{ts} \times (stage\_count - 1) \tag{5.87}$$

K_{ts}: *is a turbine stage coefficient derived from empirical data*
stage_count: is the number of turbine stages

This efficiency is also constrained:

$$\eta_{min\_turbine} \leq \eta_{turbine} \leq \eta_{max\_turbine} \tag{5.88}$$

Turbine Power and Temperature Drop Calculation In full-flow mode, the required turbine power is calculated based on the pump power and mechanical efficiency:

$$required\_turbine\_power = \frac{total\_pump\_power}{\eta_{mech}} \tag{5.89}$$

total_pump_power: is the total power required by the pump [W]
η_{mech}*: is the mechanical efficiency of the shaft connection between turbine and pump*

For multi-stage turbines, the power is distributed equally:

$$power\_per\_stage = \frac{required\_turbine\_power}{turbine\_stage\_count} \tag{5.90}$$

required_turbine_power: is the total power needed from the turbine [W]
turbine_stage_count: is the number of turbine stages

Temperature Drop Calculation:

In full-flow mode, all hot gas from the preburner passes through the turbine, and we calculate the temperature drop:

$$\Delta T = \frac{power\_per\_stage}{\dot{m}_{gas} \times c_p \times \eta_{turbine}} \tag{5.91}$$

ΔT*: is the temperature drop across the turbine stage [K]*
power_per_stage: is the power extracted by each turbine stage [W]
$\dot{m}_{gas}$*: is the gas mass flow rate [kg/s] from the preburner*
c_p*: is the specific heat capacity [J/(kg·K)] calculated from gas properties*
$\eta_{turbine}$*: is the turbine efficiency [dimensionless]*

Outlet Temperature Calculation:

The outlet temperature is determined by:

$$T_{out} = T_{in} - \Delta T \tag{5.92}$$

T_{in}*: is the inlet temperature [K] from the preburner*
ΔT*: is the temperature drop across the turbine [K]*

Net Efficiency Calculation The overall efficiency of the turbopump assembly is calculated as:

$$\eta_{net} = \eta_{pump} \times \eta_{turbine} \tag{5.93}$$

η_{pump}: is the pump efficiency [dimensionless]
$\eta_{turbine}$: is the turbine efficiency [dimensionless]

Turbopump Design Iteration To find the optimal turbopump configuration, HyperX evaluates different combinations of spool speeds and stage counts. The optimization metric is:

$$cost = \frac{1}{\eta_{net}} \tag{5.94}$$

$cost$: is the optimization metric (lower is better)
η_{net}: is the net efficiency of the turbopump assembly

The iteration process explores various spool speeds and stage counts, selecting the configuration with the minimum cost value.

Labyrinth Seal Design The labyrinth seal is crucial for preventing hot gas leakage between turbine and pump sections. Using the hot gas properties calculated earlier, the seal design parameters are determined:

$$p_{in} = turbine\_inlet\_pressure \tag{5.95}$$

p_{in}: is the seal inlet pressure [bar]
$turbine\_inlet\_pressure$: is the pressure at the turbine inlet [bar], determined by the preburner output pressure

$$p_{out} = p_{in} - pressure\_differential \tag{5.96}$$

p_{out}: is the seal outlet pressure [bar]
$pressure\_differential$: is the pressure drop across the seal [bar], determined based on sealing requirements

Tooth Clearance Calculation:

The tooth geometry parameters are defined based on engineering heuristics and material constraints:

$$clearance = min\_clearance + (teeth\_count \times clearance\_growth\_factor) \quad (5.97)$$

clearance: is the radial gap between teeth and housing [m]
min_clearance: is the minimum allowable clearance based on manufacturing tolerances [m]
teeth_count: is the number of seal teeth
clearance_growth_factor: accounts for thermal expansion and wear [m/tooth]

Swirl Break Effectiveness:

With swirl brake enabled for improved performance:

$$effective\_swirl = base\_swirl\_factor \times swirl\_brake\_effectiveness \quad (5.98)$$

effective_swirl: is the reduced swirl coefficient with brakes [dimensionless]
base_swirl_factor: is the swirl factor without brakes [dimensionless]
swirl_brake_effectiveness: is the performance improvement from swirl brakes [dimensionless]

Leakage Mass Flow Calculation:

The target leakage mass flow rate is set based on acceptable performance loss:

$$\dot{m}_{leak} = leakage\_fraction \times \dot{m}_{gas} \quad (5.99)$$

$\dot{m}_{leak}$: *is the allowable leakage flow rate [kg/s]*
leakage_fraction: is the acceptable fraction of total flow [dimensionless]
$\dot{m}_{gas}$: *is the total gas flow rate [kg/s]*

Implementation of the Full System Design To implement the complete turbopump system design, HyperX follows an iterative approach that ensures optimal performance. Turbo Pump Assembly and preburner need to work hand in hand. So an initial turbopump system design can be done with a few assumed values and then recalculate the system design parameters once the preburner system design is done.

The final design meets all performance requirements while optimizing for efficiency, reliability, and manufacturability. The system design integrates seamlessly with the preburner and main combustion chamber to create a high-performance FFSC engine.

Table 8: HyperX System Design Values For the Initial Requirement

HyperX System Design Values For the Initial Requirement		
Parameter	**Fuel Line**	**Oxidizer Line**
Total Pump Power	37 918.33 kW	42 135.50 kW
Total Turbine Power	38 692.17 kW	42 995.41 kW
Spool Speed	35 000.00 RPM	35 000.00 RPM
Net Efficiency	64.80%	64.80%

Table 9: HyperX Pump/Impeller Stage Details For the Initial Requirement

HyperX Pump/Impeller Stage Details For the Initial Requirement		
Parameter	**Fuel Line**	**Oxidizer Line**
Stage Head	62 702.16 J/kg	23 225.24 J/kg
Stage Power	12 639.44 kW	14 045.17 kW
Outlet Diameter	0.30 m	0.30 m
Blade Angle	30.00 deg	30.00 deg
Slip Factor	0.92	0.92
Flow Coefficient	0.07	0.07
Number of Blades	7	9

Table 10: HyperX Turbine Stage Details For the Initial Requirement

HyperX Turbine Stage Details For the Initial Requirement		
Parameter	**Fuel Line**	**Oxidizer Line**
Stage Power	38 692.17 kW	42 995.41 kW
Mass Flow	159.65 kg/s	384.61 kg/s
Loading Coefficient	3.00	3.00
Efficiency	90%	90%
Inlet Temperature	1843.83 K	2000 K
Outlet Temperature	1736.12 K	1950.32 K

Geometry

The advanced geometry of the HyperX fuel pump system is derived step by step using analytical formulas and conservative rocket engine heuristics. Each geometric subsystem is computed from performance outputs such as pressure rise, shaft speed, and fluid properties.

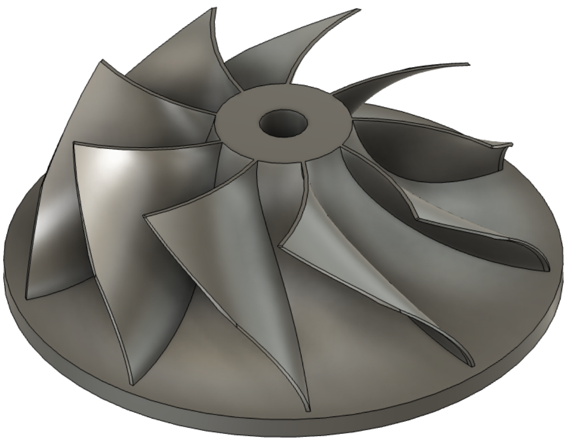

Figure 12: Turbo Pump Assembly geometric layout

Impeller Geometry The centrifugal pump impeller geometry begins with the enthalpy rise assigned to each stage. Using the Euler turbomachinery equation and a slip factor correction, the outlet tip speed of the impeller is calculated:

$$U_2 = \sqrt{\frac{h_{\text{stage}}}{\sigma}} \tag{5.100}$$

h_{stage}: is the stage-specific head [J/kg]

σ: is the slip factor (typically 0.9–0.95 for rocket-class impellers)

Angular Speed Calculation:

The angular speed ω in radians per second is computed from spool speed N in RPM:

$$\omega = \frac{2\pi N}{60} \tag{5.101}$$

Impeller Outlet Radius Calculation:

Using this, the impeller outlet radius is derived from the tip speed:

$$r_2 = \frac{U_2}{\omega} \tag{5.102}$$

In parallel, the radius is estimated from the flow coefficient relation, using the mass flow and fluid density:

$$r_{2,\text{flow}} = \left(\frac{Q}{\phi\omega}\right)^{1/3} \tag{5.103}$$

Q: is the volumetric flow rate ($\dot{m}/\rho$) [m^3/s]

ϕ: is the dimensionless flow coefficient

To ensure convergence between energy-based and flow-based estimates, the final outlet radius is averaged:

$$r_{2,\text{final}} = \frac{1}{2}(r_2 + r_{2,\text{flow}}) \tag{5.104}$$

Then the outlet diameter is:

$$d_2 = 2r_{2,\text{final}} \tag{5.105}$$

The inlet diameter is heuristically set to:

$$d_{\text{in}} = 0.4 \times d_2 \tag{5.106}$$

Inducer Geometry For cavitation suppression at the pump inlet, an axial-flow inducer is added before the impeller. The inducer has a defined tip and hub diameter. Its axial length is computed based on the radial height:

$$\text{Length}_{\text{inducer}} = 0.3 \times \left(\frac{D_{\text{tip}} - D_{\text{hub}}}{2}\right) \tag{5.107}$$

Volute Geometry The volute geometry, which recovers pressure downstream of the impeller, is defined by its throat area and spiral properties. The throat area is estimated using flow rate and approximate outlet velocity:

$$A_{\text{throat}} = \frac{Q}{v} \tag{5.108}$$

Q: is the volumetric flow rate [m^3/s]
v: is the approximate outlet velocity [m/s]

The spiral angle is typically:

$$\theta_{\text{spiral}} \approx 360^\circ \tag{5.109}$$

Casing Geometry The high-pressure casing of the pump is sized using hoop stress theory. The wall thickness is computed from the internal pressure and material strength:

$$t = \frac{P \cdot r}{\sigma_{\text{allowable}}} \tag{5.110}$$

*$\sigma_{\textbf{allowable}} = \sigma_{\textbf{yield}}/$**Safety Factor**: is the allowable stress*
r: is the internal radius [m]

A margin multiplier (e.g. 1.1) is applied to account for ports and flanges:

$$t_{\text{final}} = t \times \text{Margin Factor} \tag{5.111}$$

Shaft Geometry The shaft (spool) connecting the turbine and pump stages is designed to resist torsional stress under load. The torque is calculated from pump power and spool speed:

$$T = \frac{P_{\text{shaft}}}{\omega} \tag{5.112}$$

$P_{\textbf{shaft}}$: is the shaft power [W]
ω: is the angular velocity [rad/s]

Then, using torsion theory, the minimum shaft diameter is derived:

$$d^3 = \frac{16T}{\pi \tau_{\text{allowable}}} \tag{5.113}$$

$\tau_{allowable}$: is the allowable shear stress of the shaft material [Pa]

Finally, the shaft diameter is:

$$d = \left(\frac{16T}{\pi \tau_{\text{allowable}}} \right)^{1/3} \tag{5.114}$$

Turbine Geometry The axial turbine that powers the pump is sized to match the required shaft power. The specific heat capacity is computed from the working gas properties:

$$c_p = \frac{\gamma}{\gamma - 1} \cdot R \tag{5.115}$$

γ: is the ratio of specific heats
R: is the gas constant [J/(kg·K)]

The enthalpy drop across the turbine is:

$$\Delta h = c_p (T_{\text{in}} - T_{\text{out}}) \tag{5.116}$$

T_{in}: is the inlet temperature [K]
T_{out}: is the outlet temperature [K]

The blade tip speed is related to enthalpy drop via the stage loading coefficient:

$$U = \sqrt{\frac{2\Delta h}{\psi}} \tag{5.117}$$

ψ: is the stage loading coefficient [dimensionless]

From this, the mean rotor radius is:

$$r_{\text{mean}} = \frac{U}{\omega} \tag{5.118}$$

And the turbine inlet and tip diameters are:

$$d_{\text{inlet}} = 2r_{\text{mean}}, \quad d_{\text{tip}} = d_{\text{inlet}} + 2 \left(\frac{h_{\text{blade}}}{1000} \right) \tag{5.119}$$

h_{blade}: is the blade height [mm]

The disc thickness is scaled from a baseline:

$$t_{\text{disc}} = 12 \text{ mm} \times \text{Safety Factor} \tag{5.120}$$

And the axial stage length is estimated as:

$$L_{\text{stage}} = 0.2 \cdot r_{\text{mean}} \tag{5.121}$$

Bearings

Bearings support the high speed rotating shaft that drives the turbine and pump—impeller and inducer. Bearings ensure smooth operation, reduce friction and handle the high load with high pressure.

Thrust Bearing Thrust bearings absorb axial load along the shaft's axis. Thrust bearings counteract the axial load.

- Ball roller element load bearing or hydrostatic/hydrodynamic bearing are generally used.

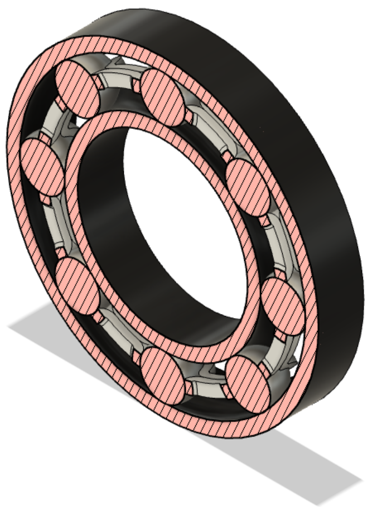

Figure 13: Thrust bearing schematic

Radial Load Bearing Radial bearings support lateral loads that are perpendicular to the shaft's axis. It helps to keep the rotating shaft stable and avoid vibrations.

- Cylindrical roller element load bearing or hydrostatic/hydrodynamic bearing are generally used.

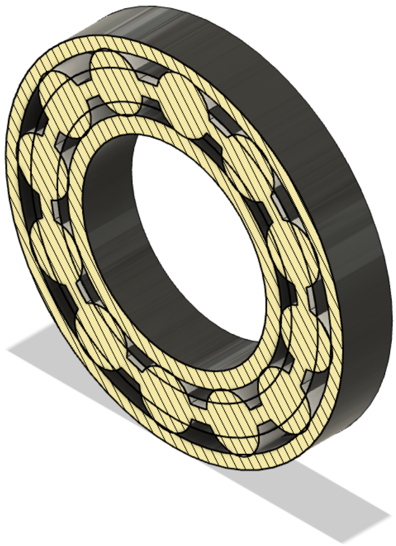

Figure 14: Radial bearing schematic

HyperX uses ball roller element bearing for thrust load and cylindrical roller element bearing for radial loads. The design of the bearing changes based on type of fuel/oxidizer and other factors. And the design might need to be changed after the CFD / additive manufacturing.

5.4 Preburner

HyperX uses two preburners where one of them is fuel-rich preburner that drives the fuel pump and another one is ox-rich preburner that drives the ox pump [1]. HyperX begins the preburner system design by operating in **full-flow mode**, where 100% of the assigned fuel and oxidizer flow through the preburner without throttling [2].

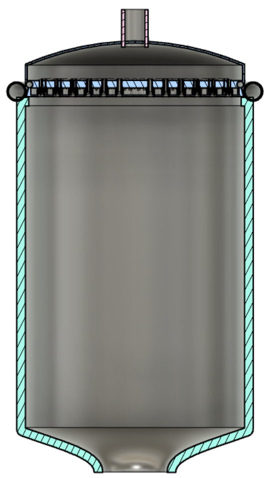

Figure 15: Preburner schematic showing fuel-rich and ox-rich configurations

5.4.1 System Design

The preburners and Turbo Pump assembly need to work hand in hand because preburner outlet gas should be able to drive the turbine to generate the power required to produce the outlet pressure target of the pump [3]. The system design parameters of the preburners are calculated in multiple steps and they are described below.

Full-Flow Staged Combustion Cycle Configuration

HyperX employs a Full-Flow Staged Combustion (FFSC) cycle where all propellants pass through the preburners before entering the main combustion chamber [1, 4]. This approach maximizes efficiency by utilizing all propellant mass for turbine power before combustion in the main chamber [1].

Oxidizer-to-Fuel Ratio Calculation The first step in the preburner design is to calculate the oxidizer-to-fuel (O/F) ratio [1]

$$O/F = \frac{\dot{m}_{ox}}{\dot{m}_{fuel}} \tag{5.122}$$

$\dot{m}_{ox}$: *is the oxidizer mass flow rate [kg/s]*
$\dot{m}_{fuel}$: *is the fuel mass flow rate [kg/s]*

For extremely low fuel flows, HyperX applies a safety constraint [5]:

$$\text{If } \dot{m}_{fuel} < 1 \times 10^{-7} \text{ then } O/F = 99.0 \tag{5.123}$$

99.0: *is a capped maximum value to avoid division by zero*
1×10^{-7}: *is the minimum threshold for fuel flow [kg/s]*

Combustion State Calculation After determining the O/F ratio, HyperX calculates the combustion state parameters [6]:

$$(T_{comb}, \gamma, R_{mix}, MW_{gmol}) = \text{computeCombustionState}(O/F, P_{pb}, f_{film}) \tag{5.124}$$

T_{comb}: *is the combustion temperature [K]*
γ: *is the specific heat ratio*
R_{mix}: *is the specific gas constant [J/kg·K]*
MW_{gmol}: *is the molecular weight [g/mol]*
O/F: *is the oxidizer-to-fuel ratio*
P_{pb}: *is the preburner pressure [bar]*

f_{film}: is the film cooling fraction

For NASA CEA calculations, HyperX first converts pressure from bar to psi [6]:

$$P_{psi} = P_{bar} \times 14.5037738 \tag{5.125}$$

P_{psi}: is the pressure in pounds per square inch
P_{bar}: is the pressure in bar
14.5037738: is the conversion factor from bar to psi

Combustion Temperature Calculation:

Next, HyperX uses NASA CEA to calculate combustion temperature [6]:

$$T_{comb-raw} = \text{CEA\_get\_Tcomb}(P_{psi}, O/F) \tag{5.126}$$

$T_{comb-raw}$: is the raw combustion temperature [K]
***CEA_get_Tcomb**: is the NASA CEA function for combustion temperature*

Molecular Weight and Specific Heat Ratio:

Then molecular weight and specific heat ratio are obtained [6]:

$$MW_{gmol}, \gamma = \text{CEA\_get\_exit\_MolWt\_gamma}(P_{psi}, O/F, \epsilon) \tag{5.127}$$

***CEA_get_exit_MolWt_gamma**: is the NASA CEA function for exit properties*
ϵ: is the expansion ratio (typically 1.05)

Molecular Weight Conversion:

Molecular weight is converted from g/mol to kg/mol:

$$MW_{kgmol} = \frac{MW_{gmol}}{1000.0} \tag{5.128}$$

MW_{kgmol}: is the molecular weight in kg/mol
1000.0: is the conversion factor from g to kg

Gas Constant Calculation:

Then specific gas constant:

$$R_{mix} = \frac{R_{univ}}{MW_{kgmol}} \tag{5.129}$$

R_{univ}: is the universal gas constant (8.314462618 J/mol·K)

Specific Heat Calculation:

HyperX determines the specific heat at constant pressure:

$$c_p = \frac{\gamma}{\gamma - 1.0} \cdot R_{mix} \tag{5.130}$$

c_p: is the specific heat at constant pressure [J/kg·K]

Empirical Fallback Calculations:

If NASA CEA is not available, HyperX uses these empirical formulas:

$$T_{comb-raw} = 1300.0 + 400.0 \cdot \left(\frac{1}{O/F + 1.0}\right) \tag{5.131}$$

1300.0*: is the base temperature [K]*
400.0*: is the temperature scaling factor [K]*

$$\gamma = 1.25 \tag{5.132}$$

1.25*: is the approximate specific heat ratio for typical rocket combustion*

$$R_{mix} = 380.0 \tag{5.133}$$

380.0*: is the approximate specific gas constant [J/kg·K]*

$$MW_{gmol} = \frac{R_{univ}}{R_{mix}} \cdot 1000.0 \tag{5.134}$$

1000.0: *is the conversion factor from kg to g*

Film Cooling Temperature Correction To protect turbine components from extreme temperatures, HyperX applies a film cooling correction [7]:

$$T_{film} = T_{comb-raw} \cdot (1.0 - f_{film} \cdot 0.7) \tag{5.135}$$

T_{film}: *is the corrected temperature after film cooling [K]*
$T_{comb-raw}$: *is the raw combustion temperature [K]*
f_{film}: *is the film cooling fraction (typically 0.05-0.07)*
0.7: *is the cooling effectiveness coefficient*

If the calculated film temperature exceeds the maximum allowable temperature [5, 7]:

$$\text{If } T_{film} > T_{max} \text{ then } T_{film} = T_{max} \tag{5.136}$$

T_{max}: *is the maximum allowable temperature [K]*

Turbine Power Calculation The turbine power is calculated using the enthalpy drop method. First, HyperX calculates the total mass flow [3]:

$$\dot{m}_{total} = \dot{m}_{fuel} + \dot{m}_{ox} \tag{5.137}$$

$\dot{m}_{total}$: *is the total mass flow through the preburner [kg/s]*

Temperature Drop Calculation:

Next, the temperature drop across the turbine is determined [3]:

$$\Delta T = T_{in} - T_{out} \tag{5.138}$$

ΔT: *is the temperature drop [K]*
T_{in}: *is the turbine inlet temperature ($= T_{film}$) [K]*
T_{out}: *is the turbine outlet temperature [K]*

HyperX ensures the temperature drop is non-negative:

$$\text{If } \Delta T < 0.0 \text{ then } \Delta T = 0.0 \tag{5.139}$$

Turbine Power Calculation:

Finally, the turbine power is calculated [3]:

$$P_{turbine} = \dot{m}_{total} \cdot c_p \cdot \Delta T \cdot \eta_{turbine} \tag{5.140}$$

$P_{turbine}$: is the turbine power [W]
$\eta_{turbine}$: is the turbine efficiency (typically 0.90-0.95)

Spool Speed Calculation HyperX converts spool speed from radians per second to revolutions per minute [3]:

$$\omega_{rpm} = \omega_{rad/s} \cdot \frac{60}{2\pi} \tag{5.141}$$

ω_{rpm}: is the rotational speed in revolutions per minute [RPM]
$\omega_{rad/s}$: is the rotational speed in radians per second [rad/s]
60: is the conversion factor from seconds to minutes
2π: is the conversion factor from radians to revolutions

Table 11: HyperX Preburner System Design Values for the Initial Requirement

HyperX Preburner System Design Values for the Initial Requirement		
Parameter	**Fuel Rich Preburner**	**Ox Rich Preburner**
Chamber Pressure	810 bar	810 bar
Chamber Temperature	1843.83 K	2000.00 K
Fuel Flow Rate	116.11 kg/s	14.51 kg/s
Oxidizer Flow Rate	43.54 kg/s	370.10 kg/s
O/F Ratio	0.38	25.50

Geometry

This section describes the geometry design of these preburners, focusing on how HyperX determines the critical dimensions of the preburner components to ensure optimal performance and structural integrity [8].

Preburner Design Overview HyperX preburners operate in full-flow mode, where all propellants flow through the preburners before entering the main combustion chamber [4]. The geometry of the preburners is designed to accommodate the high-pressure, high-temperature environment while ensuring efficient partial combustion [7, 8].

Injector Head And Film Cooling Flow Calculation HyperX divides the total propellant flow into a main combustion stream and a film cooling stream to protect the chamber walls. Main combustion stream reaches the injector head and the film cooling stream reaches the film cooling nozzles on the walls of the chamber cylinder [9, 10].

$$\dot{m}_{film} = \dot{m}_{total} \times f_{film} \tag{5.142}$$

$\dot{m}_{film}$: is the mass flow for film cooling [kg/s]
$\dot{m}_{total}$: is the total preburner mass flow [kg/s]
f_{film}: is the film cooling fraction (typically 0.05-0.07)

$$\dot{m}_{swirl} = \dot{m}_{total} - \dot{m}_{film} \tag{5.143}$$

$\dot{m}_{swirl}$: is the remaining flow for swirl-based combustion [kg/s]
[9]

The injector head is designed similar to the main combustion chamber injector head with coaxial swirl injectors and dome manifold [9, 10]. One of the fluids in the bi-propellant design enters via the top of the manifold dome and the other enters on the side. The individual coaxial swirl injectors are arranged in a ring format [9]. The initial setup can be assumed like [6, 12, 18] which gives 36 injectors. Again this ring setup can be iterated based on the mass flow per injector and total mass flow of the preburner.

Chamber Diameter Calculation HyperX first calculates the chamber diameter based on the target mass flux through the preburner. Starting with a base area derived from the total mass flow and target mass flux [1, 8]:

$$A = \frac{\dot{m}_{total}}{G} \tag{5.144}$$

A: is the cross-sectional area [m²]
$\dot{m}_{total}$: is the total mass flow rate through the preburner [kg/s]

G: is the target mass flux [kg/(m²·s)]

Base Diameter Calculation:

The base diameter is then calculated from this area [8]:

$$d_{base} = \sqrt{\frac{4A}{\pi}} \tag{5.145}$$

d_{base}: *is the base chamber diameter [m]*
A: *is the cross-sectional area*

Final Chamber Diameter:

The final chamber diameter is obtained by applying a scaling factor [8]:

$$d_{chamber} = d_{base} \times d_{ratio} \tag{5.146}$$

$d_{chamber}$: *is the final chamber diameter [m]*
d_{base}: *is the base diameter*
d_{ratio}: *is the chamber diameter ratio (typically 1.2-1.3)*

Chamber Length Calculation HyperX determines the chamber length by considering both swirl-based mixing requirements and geometric proportions. First, the effective swirl mixing length is calculated [8, 9]:

$$L_{swirl,eff} = F_{swirl} \times L_{swirl} \tag{5.147}$$

$L_{swirl,eff}$: *is the effective swirl mixing length [m]*
F_{swirl}: *is the swirl length factor (typically 3.0)*
L_{swirl}: *is the basic swirl mixing length [m]*

Length-to-Diameter Ratio Calculation:

Separately, a length based on the chamber diameter and a length-to-diameter ratio is calculated [8]:

$$L_{ratio} = d_{chamber} \times \left(\frac{L}{d}\right) \tag{5.148}$$

L_{ratio}: *is the length based on the diameter ratio [m]*
$d_{chamber}$: *is the chamber diameter*
(L/d): *is the length-to-diameter ratio (typically 1.4-1.6)*

Final Chamber Length:

Then the maximum of these two lengths and a dome shape correction factor is applied [8]:

$$L_{chamber} = \max(L_{swirl,eff}, L_{ratio}) \times F_{dome} \tag{5.149}$$

$L_{chamber}$: *is the final chamber length [m]*
$L_{swirl,eff}$: *is the effective swirl mixing length*
L_{ratio}: *is the length based on the diameter ratio*
F_{dome}: *is the dome shape factor (typically 1.05)*

Wall Thickness Calculation The chamber wall thickness is designed to handle the high internal pressure. First, HyperX applies a temperature-based derating to the material yield strength if the operating temperature exceeds 700 K [11, 12]:

$$\sigma_{yield,derated} = 0.7 \times \sigma_{yield} \tag{5.150}$$

$\sigma_{yield,derated}$: *is the derated yield strength [Pa]*
σ_{yield}: *is the material yield strength at room temperature [Pa]*
0.7: *is the derating factor for high-temperature operation*

Wall Thickness from Hoop Stress:

Then, the wall thickness is calculated using the hoop stress equation [11]:

$$t = \frac{P \times r}{\sigma_{yield,derated} \times SF} \tag{5.151}$$

t: *is the wall thickness [m]*
P: *is the design pressure [Pa]*
r: *is the chamber radius [m]*
$\sigma_{yield,derated}$: *is the derated yield strength*
SF: *is the safety factor (typically 1.4-1.5)*

The wall thickness is converted to millimeters for practical application [11]:

$$t_{mm} = t \times 1000 \tag{5.152}$$

t_{mm}: is the wall thickness [mm]
t: is the wall thickness [m]

Nozzle Geometry Calculation HyperX designs the convergent nozzle section that connects the preburner to the turbine. The nozzle length is proportional to the chamber diameter. This nozzle is not as big as the main combustion chamber nozzle but to expand the flow slightly [1, 8].

$$L_{nozzle} = d_{chamber} \times L_{ratio,nozzle} \tag{5.153}$$

L_{nozzle}: is the nozzle convergent length [m]
$d_{chamber}$: is the chamber diameter
$L_{ratio,nozzle}$: is the nozzle length ratio (typically 0.25)

Throat Diameter Calculation:

The nozzle throat diameter can be determined in three ways. If computed from mass flux [1]:

$$d_{throat} = \sqrt{\frac{4\dot{m}_{total}}{\pi G_{nozzle}}} \tag{5.154}$$

d_{throat}: is the nozzle throat diameter [m]
$\dot{m}_{total}$: is the total preburner mass flow [kg/s]
G_{nozzle}: is the target mass flux at the throat (typically 1000 kg/(m²·s))

Alternatively, a simplified approach uses a fraction of the chamber diameter [1]:

$$d_{throat} = 0.8 \times d_{chamber} \tag{5.155}$$

0.8: is a typical scale factor for the throat-to-chamber diameter ratio

Volume and Surface Area Calculation HyperX calculates the internal volume of the preburner chamber, including any additional volume for spool [8]:

$$V_{chamber} = \pi r^2 L_{chamber} + V_{spool\_mount} \tag{5.156}$$

$V_{chamber}$: is the total chamber volume [m³]
r: is the chamber radius [m]
$L_{chamber}$: is the chamber length
V_{spool_mount}: is the additional volume for spool mounting [m³]

The external surface area is calculated for thermal analysis [8]:

$$A_{surface} = 2\pi r L_{chamber} + \pi r^2 \tag{5.157}$$

$A_{surface}$: is the external surface area [m²]
r: is the chamber radius [m]
$L_{chamber}$: is the chamber length

Specific Preburner Configurations **Fuel-Rich Preburner:**

For the fuel-rich preburner, HyperX uses the following key parameters:

$$O/F_{fr} = 0.5 \tag{5.158}$$

$$f_{film-fr} = 0.07 \tag{5.159}$$

$$d_{ratio-fr} = 1.2 \tag{5.160}$$

$$(L/d)_{fr} = 1.6 \tag{5.161}$$

O/F_{fr}: is the oxidizer-to-fuel ratio for the fuel-rich preburner
$f_{film-fr}$: is the film cooling fraction for the fuel-rich preburner
$d_{ratio-fr}$: is the chamber diameter ratio for the fuel-rich preburner
$(L/d)_{fr}$: is the length-to-diameter ratio for the fuel-rich preburner

Ox-Rich Preburner:

For the ox-rich preburner, HyperX uses these parameters:

$$O/F_{or} = 25.0 \tag{5.162}$$

$$f_{film-or} = 0.05 \tag{5.163}$$

$$d_{ratio-or} = 1.3 \tag{5.164}$$

$$(L/d)_{or} = 1.4 \tag{5.165}$$

O/F_{or}*: is the oxidizer-to-fuel ratio for the ox-rich preburner*
$f_{film-or}$*: is the film cooling fraction for the ox-rich preburner*
$d_{ratio-or}$*: is the chamber diameter ratio for the ox-rich preburner*
$(L/d)_{or}$*: is the length-to-diameter ratio for the ox-rich preburner*

For the ox-rich preburner, an additional oxygen corrosion factor is applied to the wall thickness:

$$t_{or} = \frac{P \times r}{\sigma_{yield,derated} \times SF} \times \frac{1}{F_{corr}} \tag{5.166}$$

t_{or}*: is the wall thickness for the ox-rich preburner [m]*
F_{corr}*: is the oxygen corrosion factor (typically 1.07)*

6 Conclusion

The theoretical design of the HyperX Full-Flow Staged Combustion Engine represents a comprehensive approach to modern rocket propulsion system development. Through systematic application of fundamental thermodynamic principles, fluid dynamics, and mechanical design concepts, we have established a robust framework for translating theoretical rocket engine design into a practical 3D model. The HyperX engine, with its target 1800 kN thrust and 300 bar chamber pressure, exemplifies how methodically derived design parameters can inform the creation of complex propulsion systems.

This paper has demonstrated the critical interdependencies between various engine components—from the main combustion chamber and nozzle to the turbopumps and preburners—and how their individual designs must be harmonized to achieve optimal system performance. The full-flow staged combustion cycle, while complex, offers significant efficiency advantages that justify its selection for high-performance applications. By utilizing liquid methane and liquid oxygen propellants, the HyperX design also aligns with the industry trend toward cleaner, more sustainable propulsion technologies.

It is important to emphasize that the theoretical design presented herein serves as the foundation for subsequent development phases. The next stages would involve computational fluid dynamics analysis to refine flow characteristics, finite element analysis to validate structural integrity, and iterative optimization through additive manufacturing trials. While theoretical models provide essential groundwork, the practical implementation will inevitably require refinements based on empirical data and testing results.

The methodologies outlined in this paper not only contribute to the specific development of the HyperX engine but also offer valuable insights for rocket propulsion engineers working on similar high-performance systems. By documenting the design process from fundamental equations to component geometry specifications, we hope to advance the field's collective understanding of modern rocket engine design and contribute to the ongoing evolution of space propulsion technology.

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